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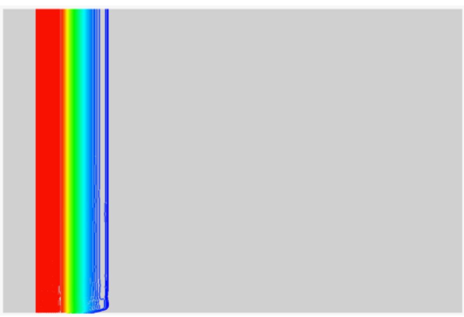
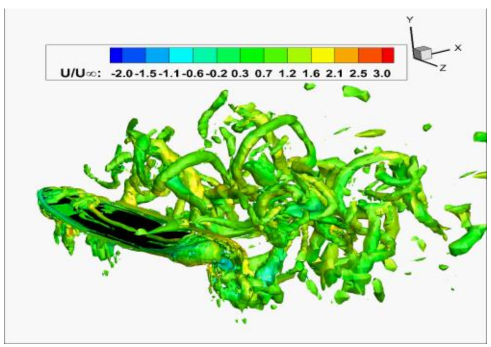
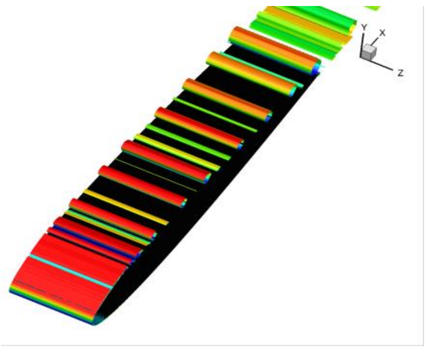
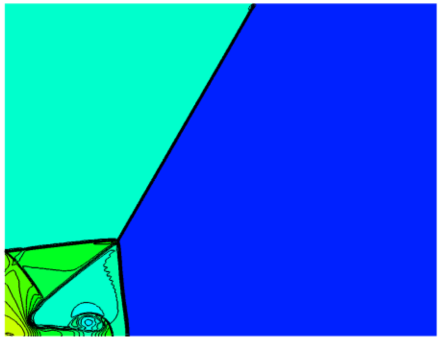
High-Order Numerical Simulation of Shock/Boundary Layer Interaction over Surface Roughness Using the FR/CPR-LLAV Method









Modeling Methods Session, TFAWS (NASA)

Meilin Yu, Assistant Professor
Department of Mechanical Engineering
University of Maryland, Baltimore County

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- Background
- Numerical Methods & Verification
- Shock/Boundary Layer Interaction (SBLI)
  - SBLI over smooth walls
  - SBLI over surface roughness
- Conclusions & Future Work

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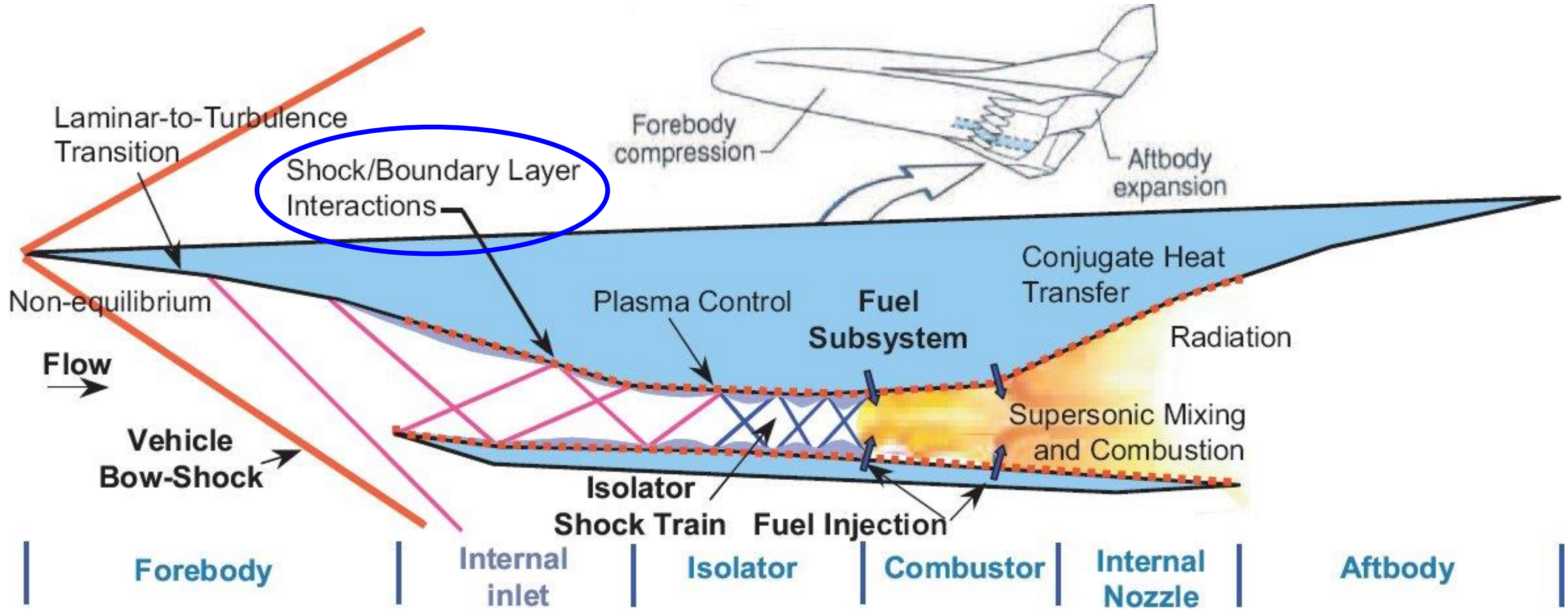
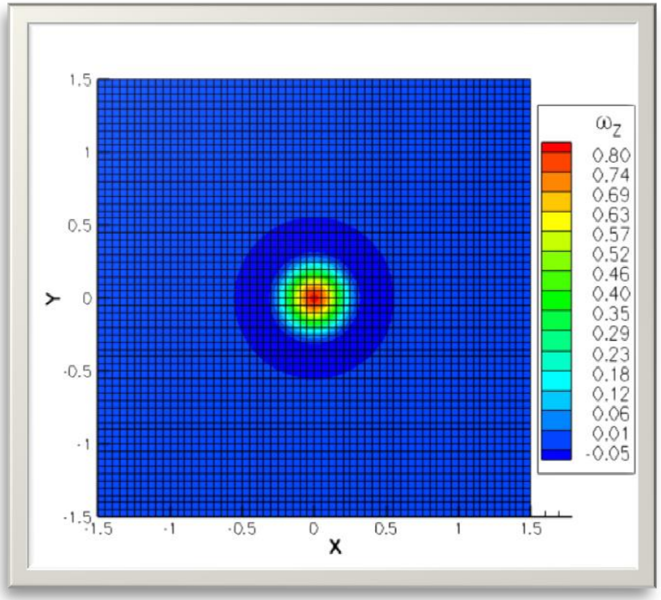
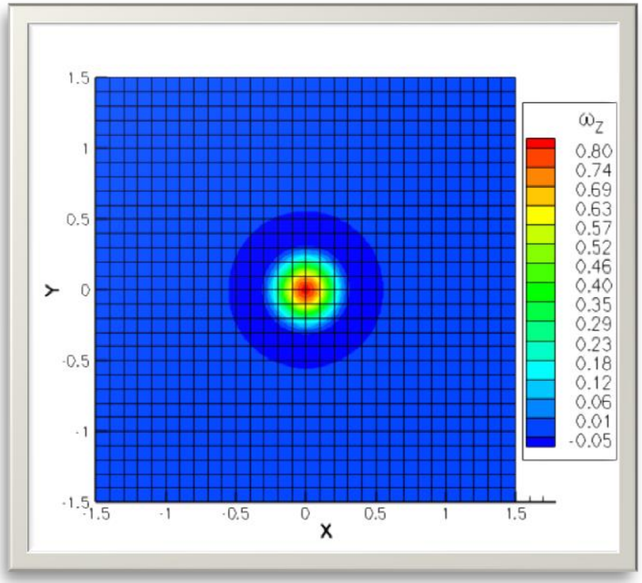
Background



Schematics of fluid flow and heat transfer in a scramjet engine
(Source: FPCE group, Stanford University)

Why High-Order CFD Methods?





2nd order 4th order

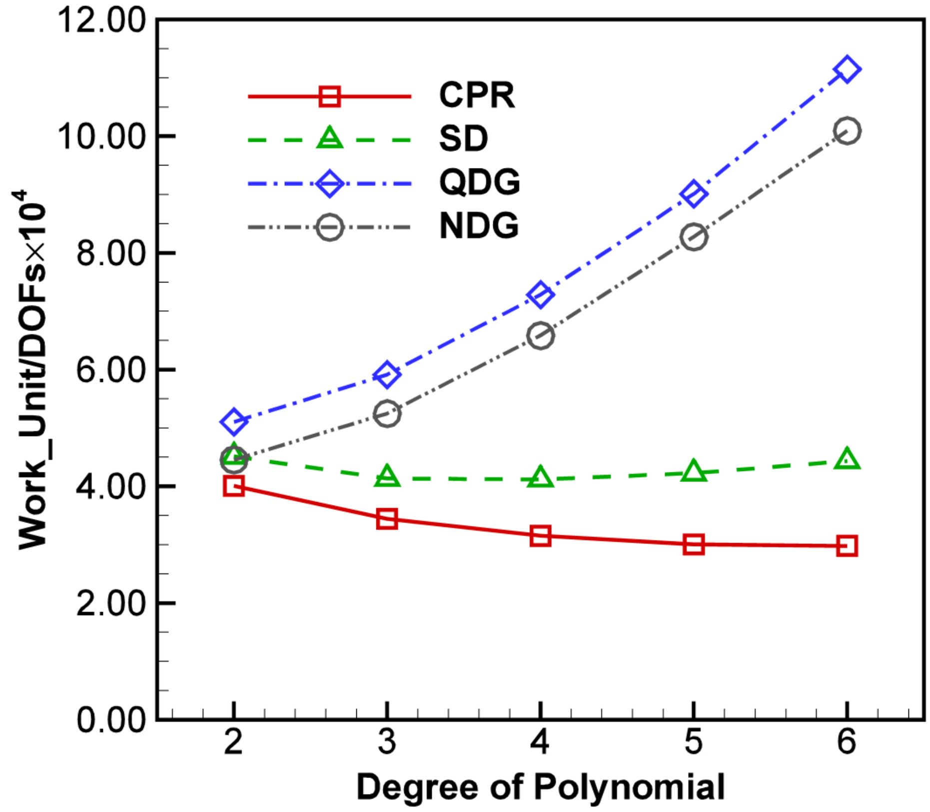
DOFs* for the 2nd order scheme: (60×60)×4=14400

DOFs for the 4th order scheme: (30×30)×16=14400

Euler vortex propagation

*: DOFs is short for degrees of freedom

Computational Cost of High-Order Methods



Computational cost per degree of freedom vs. polynomial order
Euler vortex propagation, linear elements

Objectives

- Develop robust localized Laplacian artificial viscosity (LLAV) shock capturing procedures for the high-order flux reconstruction/correction procedure via reconstruction (FR/CPR) method

- Explore flow physics of complex shock-boundary layer interaction over surface roughness with the high-order FR/CPR-LLAV method

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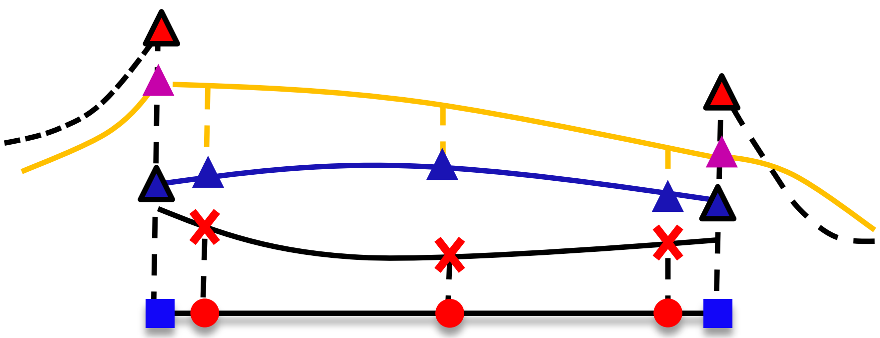
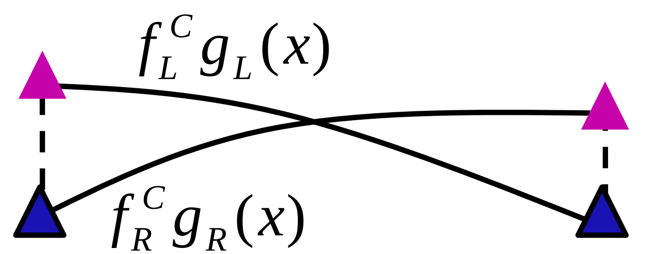
Flux Reconstruction/Correction Procedure via Reconstruction (FR/CPR)

- First developed by H. T. Huynh (2007)

$$\frac{\partial Q}{\partial t}+\frac{\partial f}{\partial x}=0 \longrightarrow \frac{\partial Q_h}{\partial t}+\frac{\partial f_h^I}{\partial x}=0,\quad Q_h \in P^k(\Omega),\quad f_h^I \in P^{k+1}(\Omega)$$

$$\frac{\partial f_h^I}{\partial x}=\frac{\partial\left(f_h^D+f_h^C\right)}{\partial x},\quad f_h^D \in P^k(\Omega),\quad f_h^C \in P^{k+1}(\Omega)$$





$$\boxed{f_h^C(x)=f_L^C g_L(x)+f_R^C g_R(x)}$$

- Very efficient high-order algorithm
- Generalization of discontinuous Galerkin and spectral difference/volume
- Compact, suitable for parallel computation

Localized Laplacian Artificial Viscosity

$$\boxed{\frac{\partial Q}{\partial t} + \nabla \cdot \boldsymbol{F}^{inv}(Q) = \nabla \cdot \boldsymbol{F}^{av}(Q, \nabla Q)}$$

Laplacian: $\boldsymbol{F}^{av}(Q, \nabla Q) = \varepsilon \nabla Q$

For each element e:

$$\varepsilon_e = \begin{cases} 0 & if\ S_e < S_0 - \kappa \\ \frac{\varepsilon_0}{2}\left(1 + sin\frac{\pi(S_e - S_0)}{2\kappa}\right) & if\ S_0 - \kappa \le S_e \le S_0 + \kappa \\ \varepsilon_0 & if\ S_e > S_0 + \kappa. \end{cases}$$

Parameters in ε_e:

$\varepsilon_0 = f(\Delta\xi_{max}, Pe) \cdot h \cdot |\lambda|_{max}$, based on the definition of the $P\acute{e}clet$ number Pe for a diffusion process:

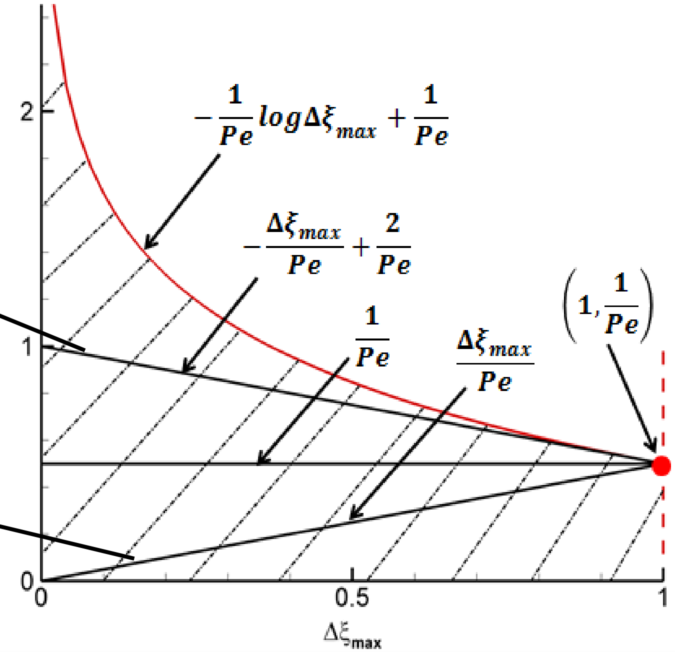
Resolution-based smoothness indicator:

$$S_e = log_{10}\frac{\langle Q - Q^{proj}, Q - Q^{proj}\rangle_e}{\langle Q, Q\rangle_e},$$

Localized Laplacian Artificial Viscosity (Cont.)

Adopted in current study

P.-O. Persson & J. Peraire (2006)

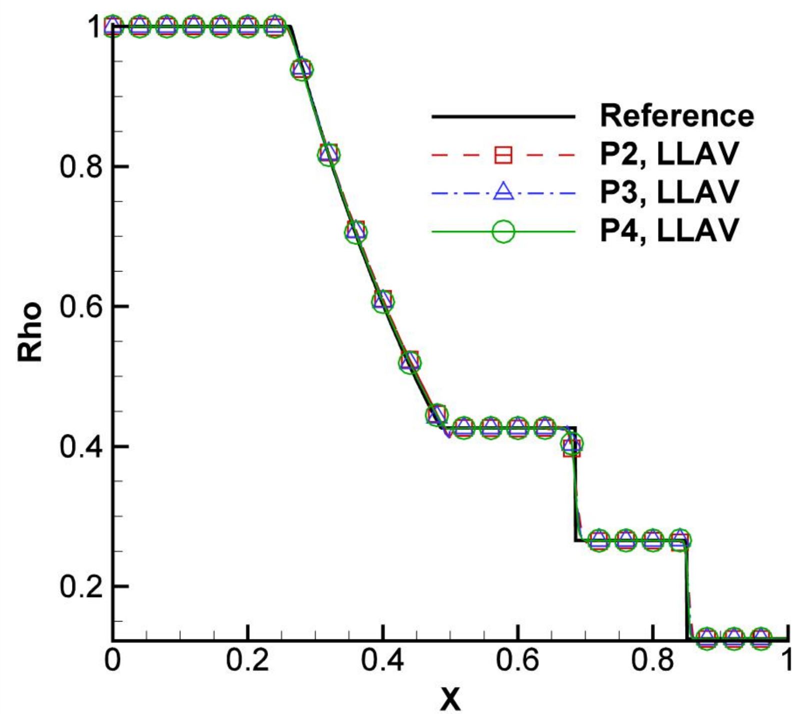


Modeling criteria:

- The artificial viscosity ε_0 is non-negative;
- When the resolution of the numerical scheme is infinite, the artificial viscosity $\varepsilon_0 \to 0$;
- The modeling is compatible with the classic results from the 2nd order accurate (or equivalently P^1 reconstruction) methods.

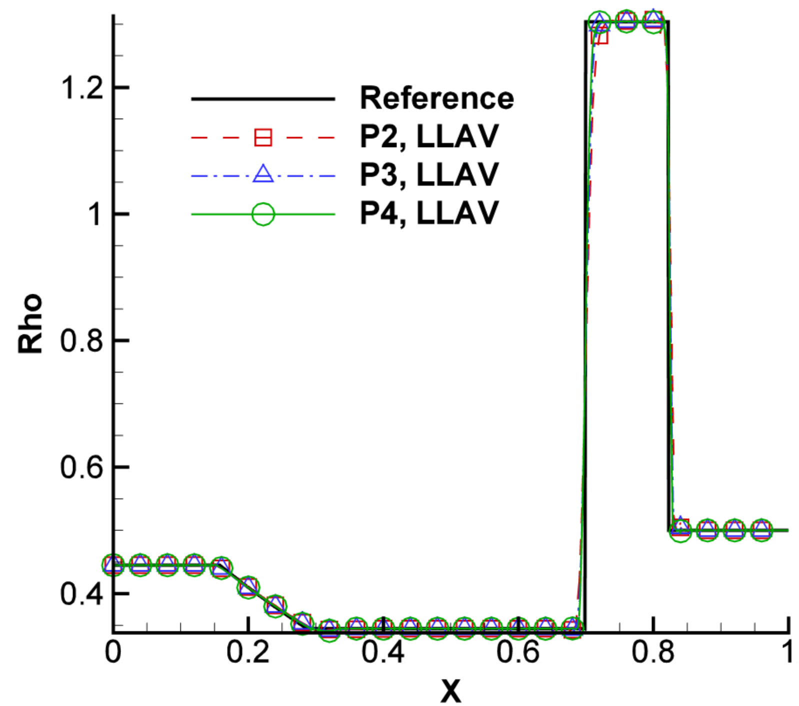
1D Shock Tube Problem

Sod Problem



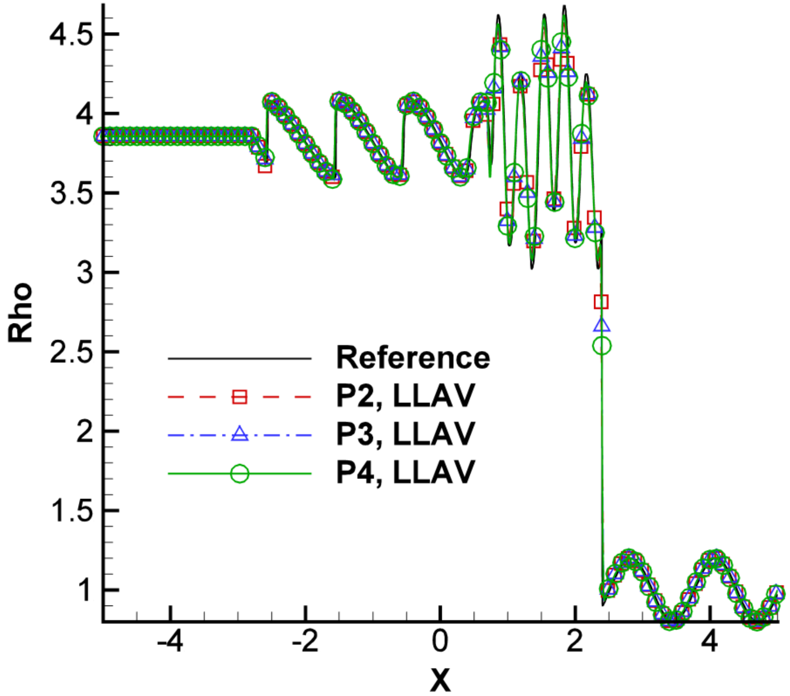
Density distribution at $t = 0.2s$

Harten-Lax Problem

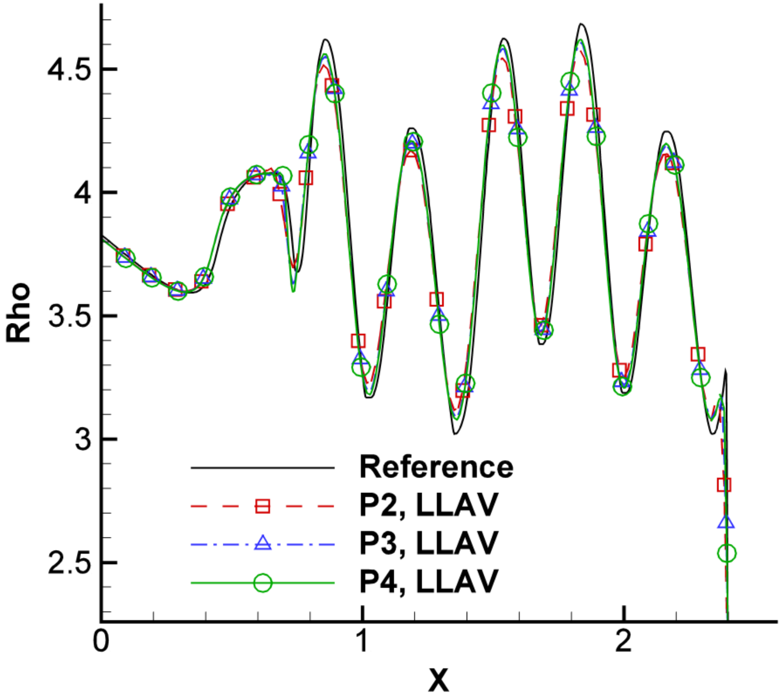


Density distribution at $t = 0.13s$

Shu-Osher Problem



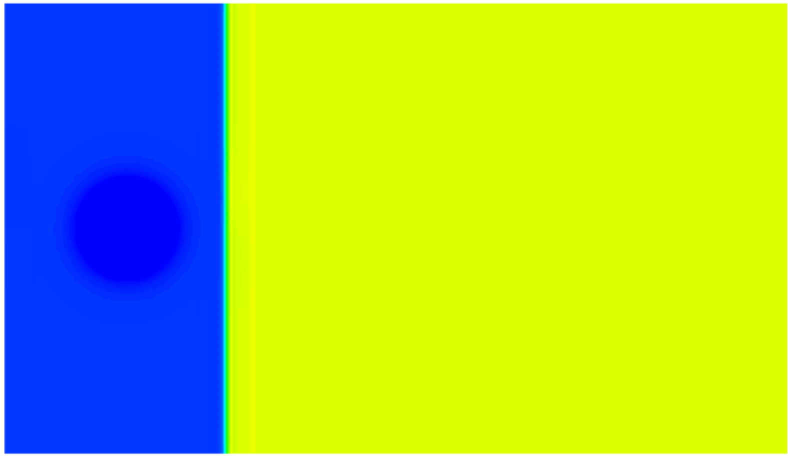
Overview



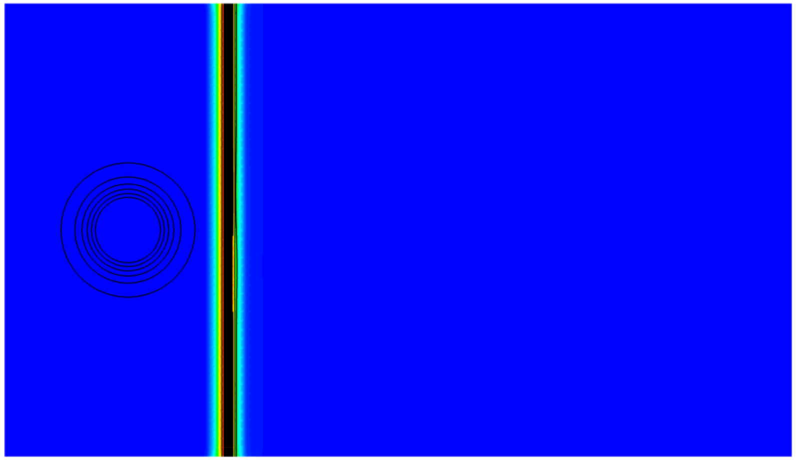
Close-up view

Density distribution at t = 1.8s

Shock-Isentropic Vortex Interaction



Pressure



Artificial viscosity

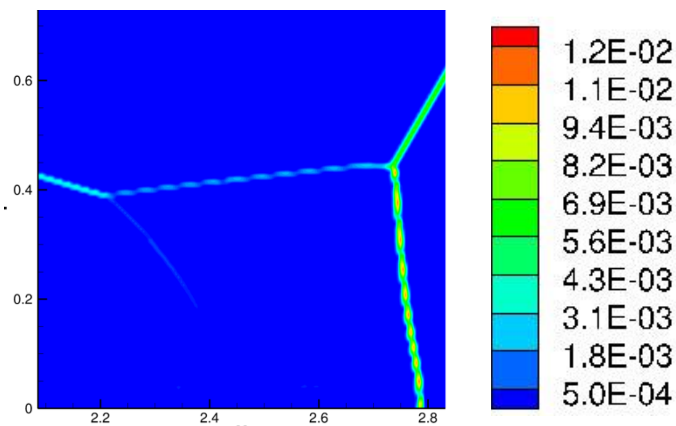
Free stream Ma =1.1,
P^3 reconstruction (4th order),
Computational domain:
$[0,2] \times [0,1]$,
100×50 elements.

An isentropic vortex is superposed to the supersonic flow.

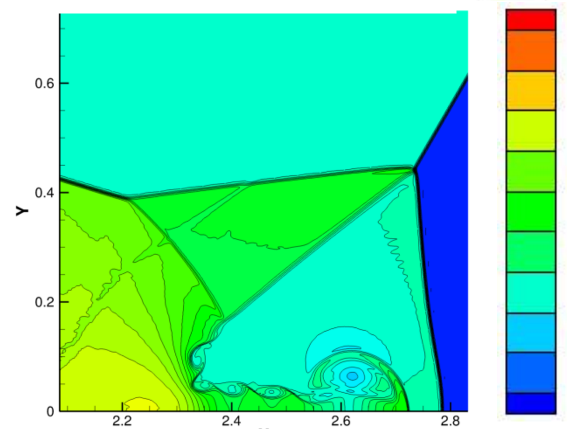
Double Mach Reflection

Density

Artificial viscosity at t=0.2s



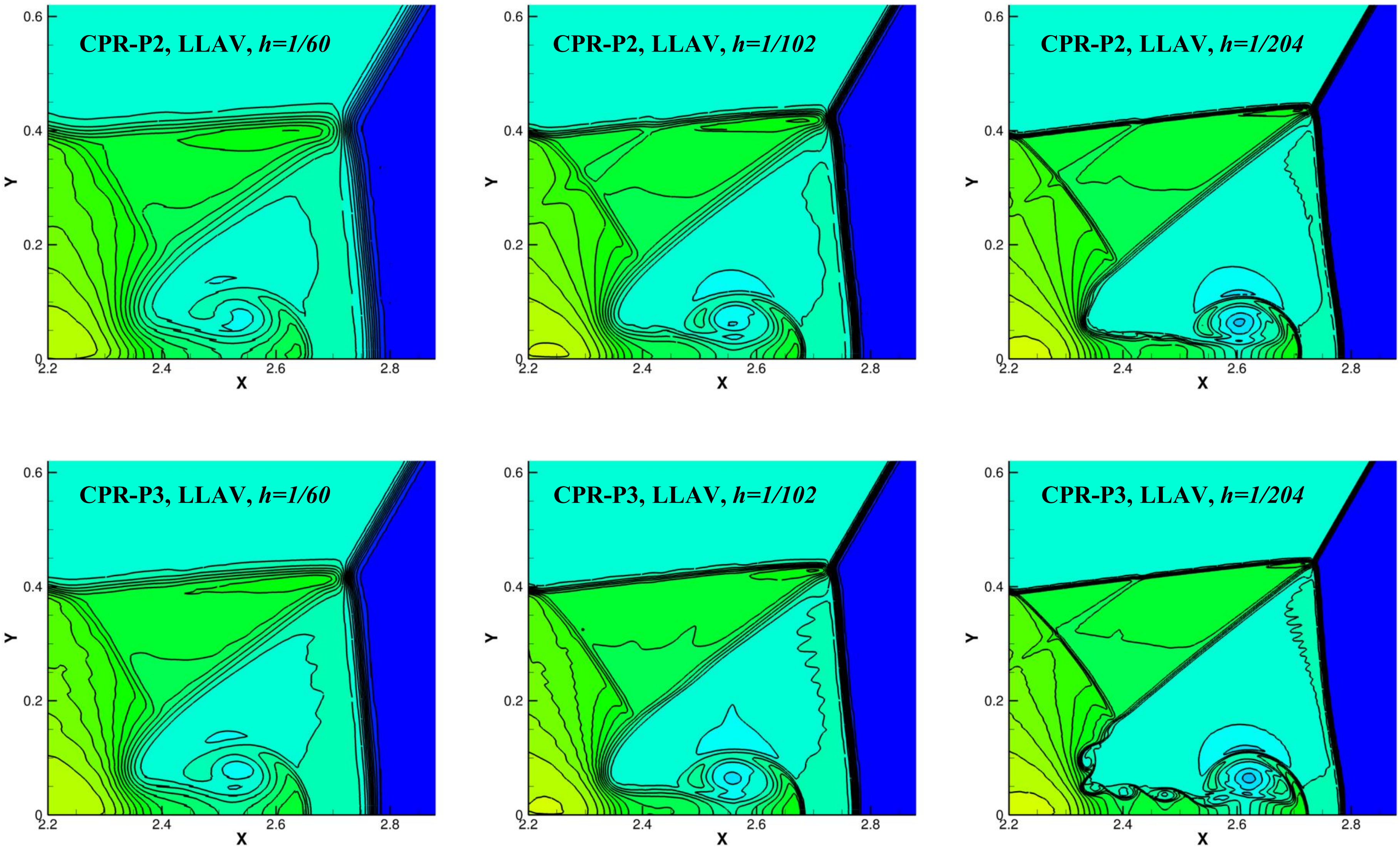
Density at t=0.2s



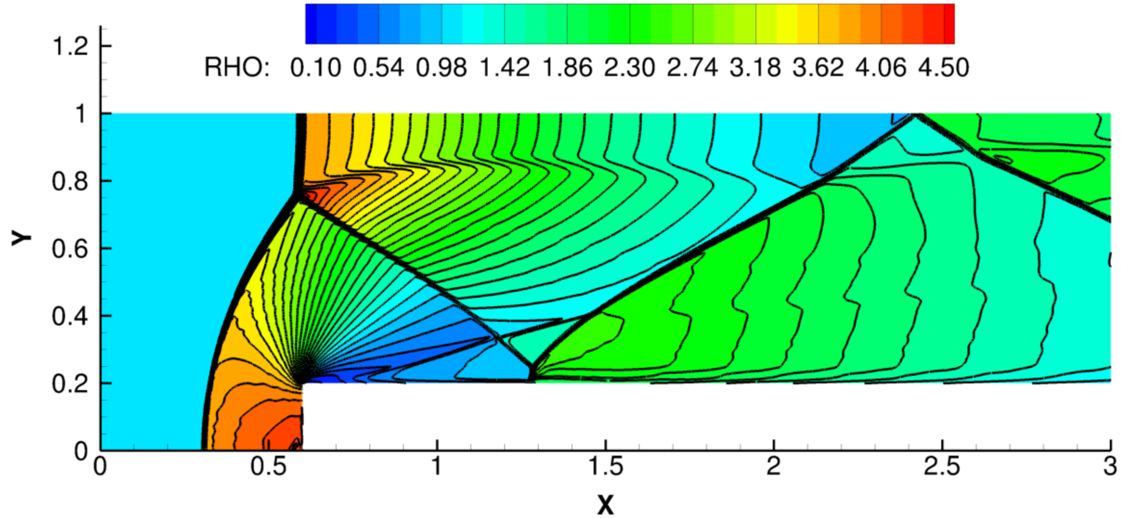
Ma =10, P^3 reconstruction (4th order), $t \in [0, 0.2s]$

Computational domain $[0,4] \times [0,1]$, 816×204 elements

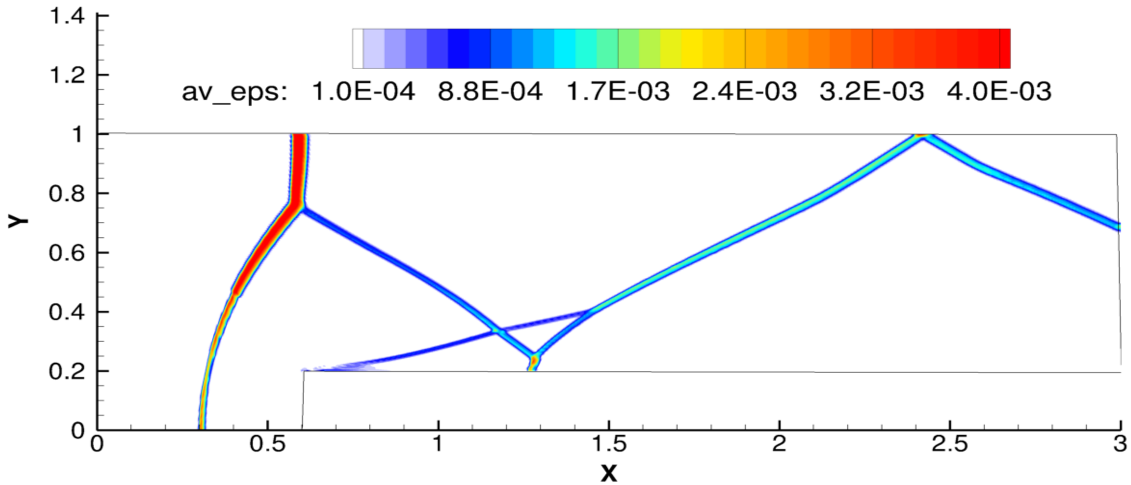
Double Mach Reflection (Cont.)



Ma 3 Wind Tunnel with a Forward Facing Step



Density at t=4s



Artificial viscosity at t=4s

Free stream Ma =3,
P^2 reconstruction (3rd order),
Grid size: 1/80, with clustered elements of size 1/320 near the sharp corner.

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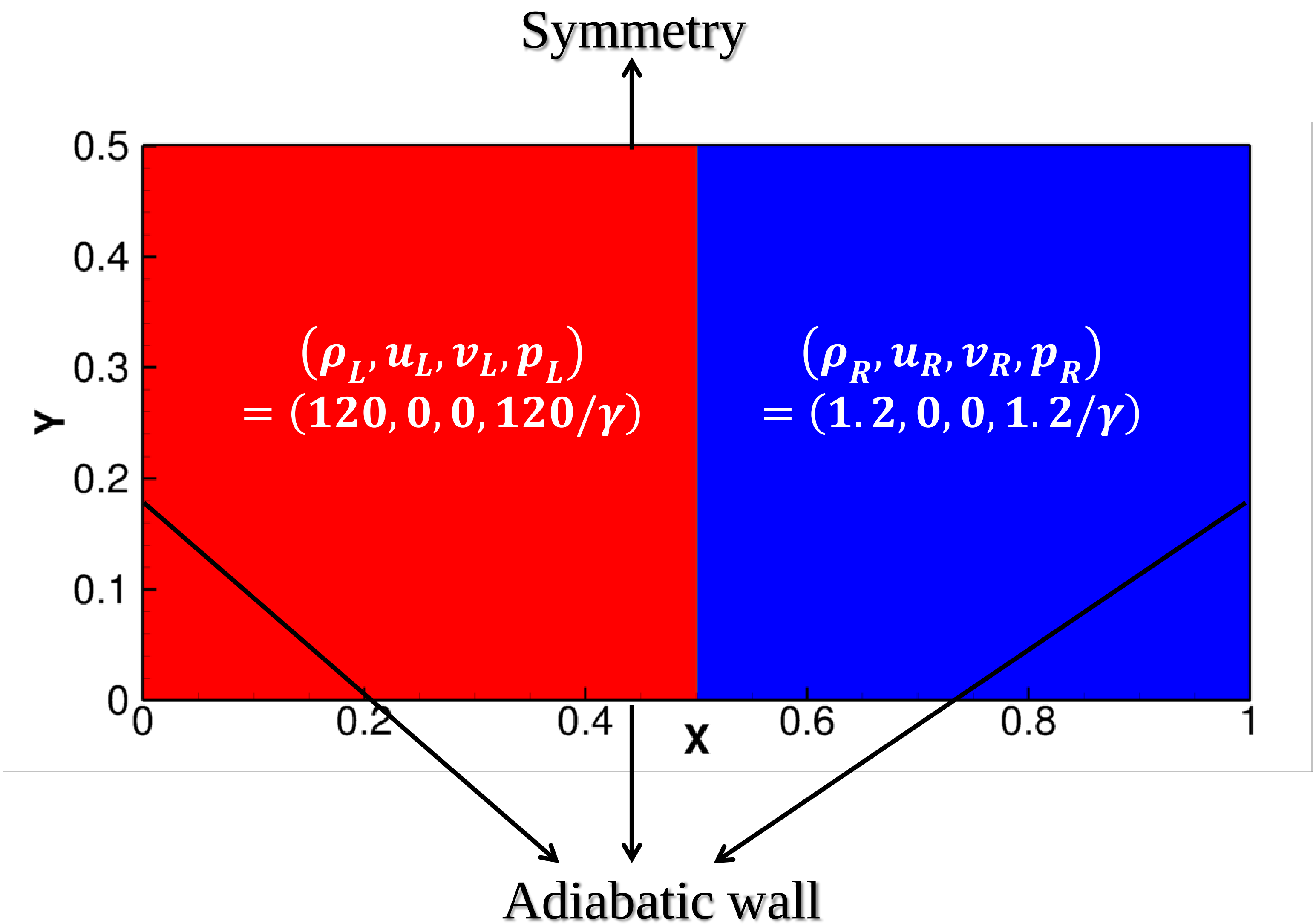
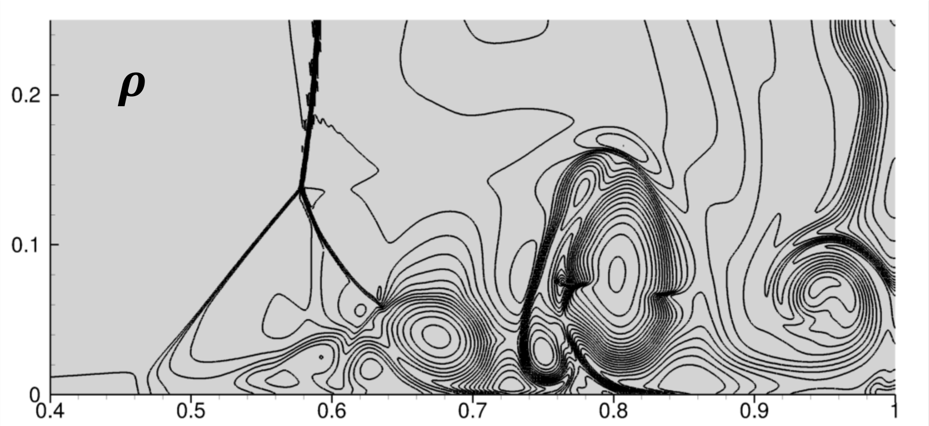
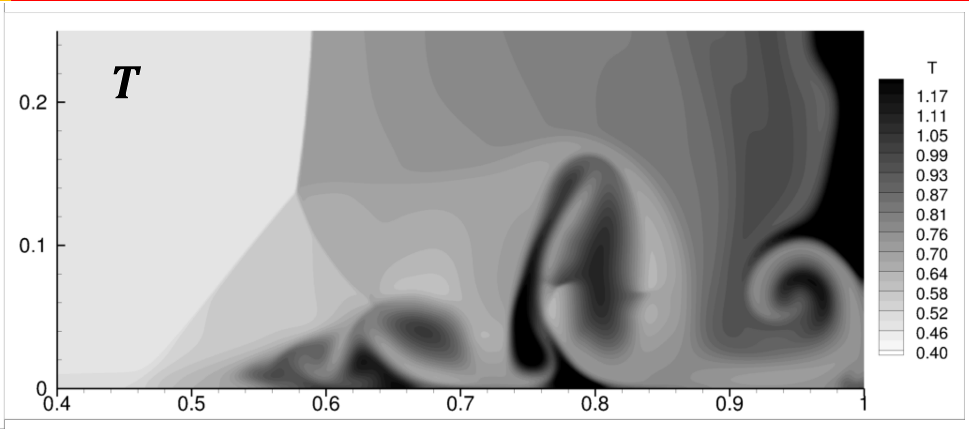
Initial & Boundary Conditions

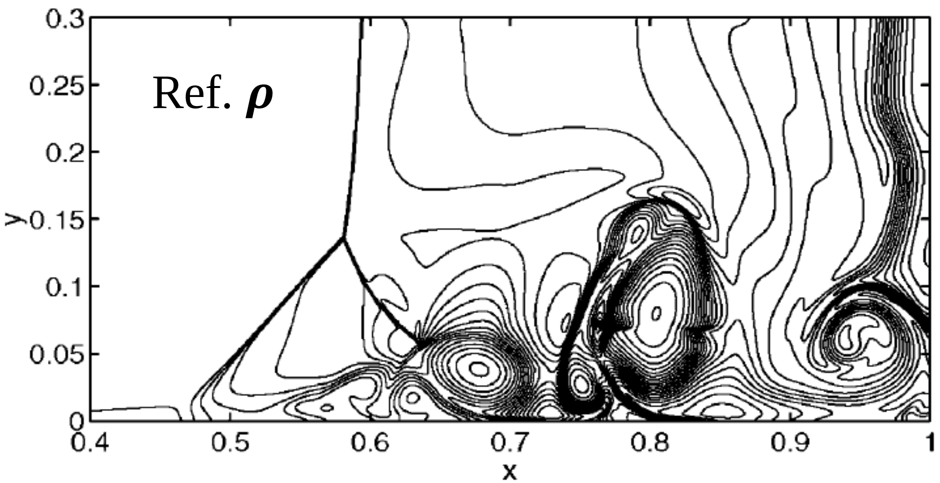


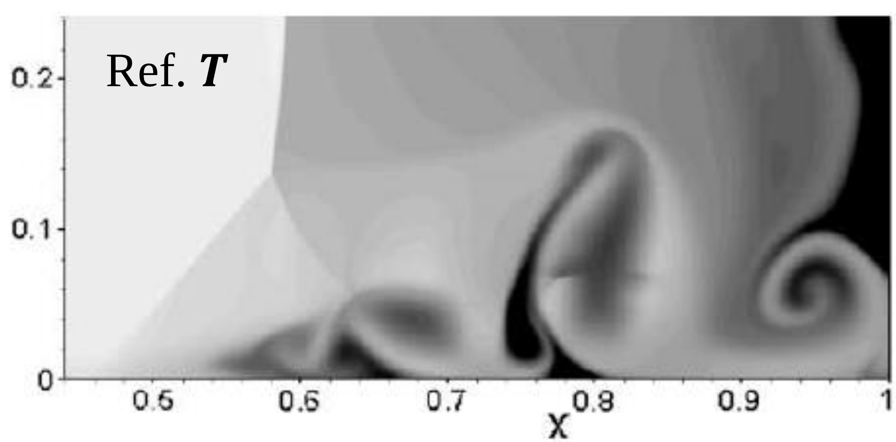
SBLI over Smooth Walls at Re=200





P^2 reconstruction on a 500×250 mesh (1.125×10^6 DOFs)

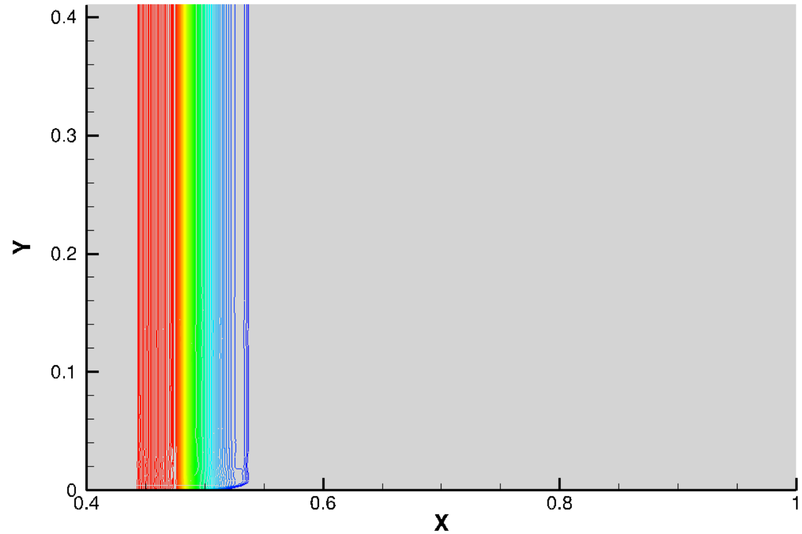




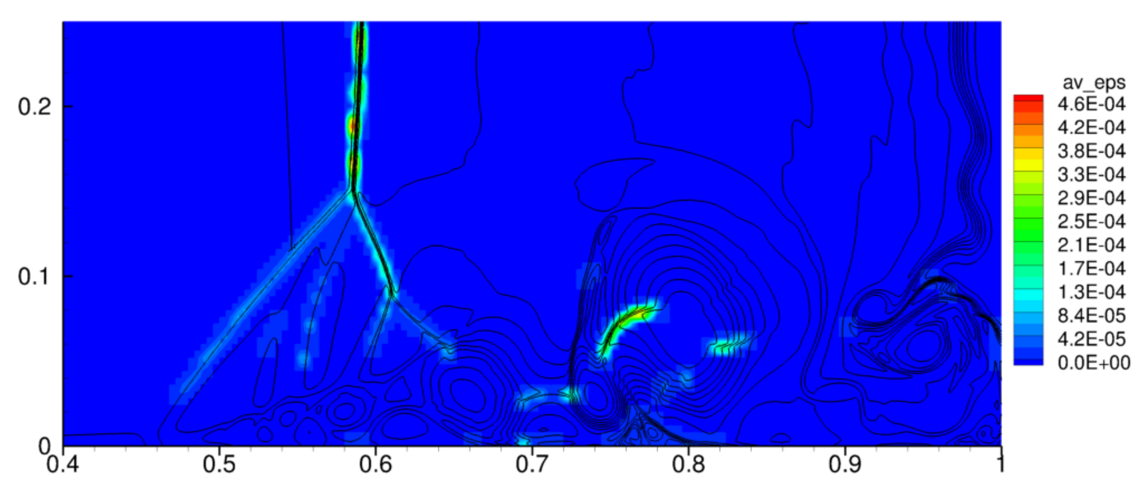
2nd order MUSCL scheme on a 3000×1500 mesh (4.5×10^6 DOFs) (Sjogreen & Yee, 2003)

7th order FD scheme on a 1000×500 mesh (0.5×10^6 DOFs) (Daru & Tenaud, 2009)

SBLI over Smooth Walls at Re=1,000

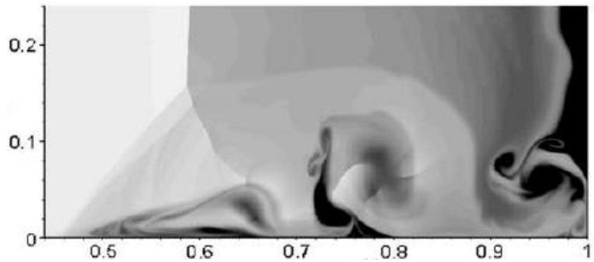


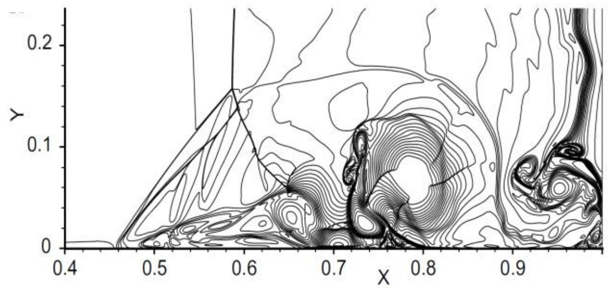
Density field



Distribution of LLAV

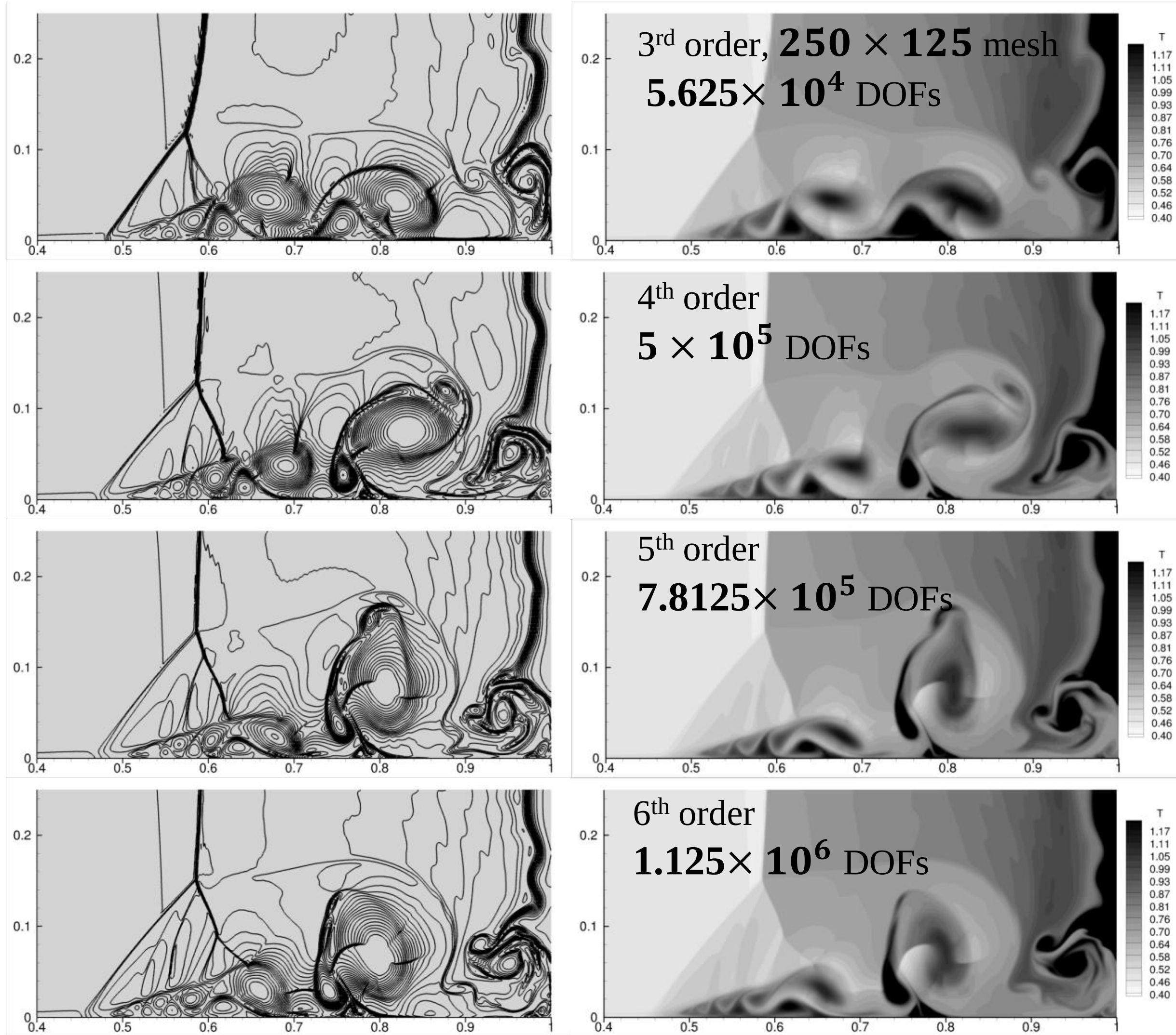
SBLI over Smooth Walls at Re=1,000 (Cont.)





7^{th} order FD scheme on a 4000×2000 mesh (8×10^6 DOFs)

(Daru & Tenaud, 2009)



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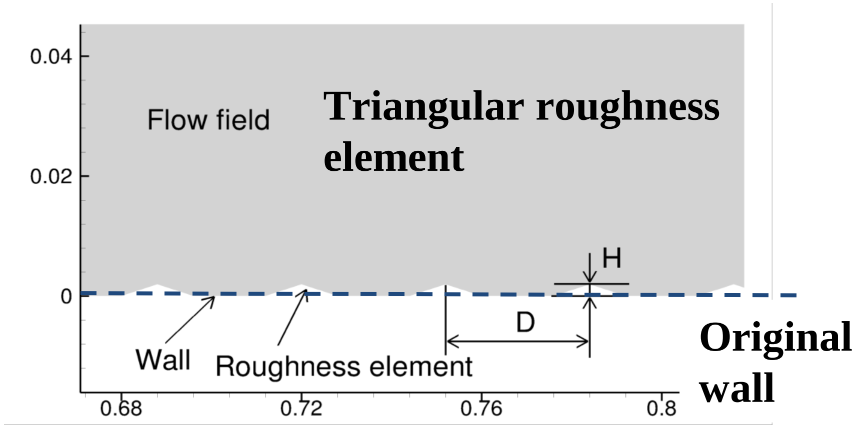
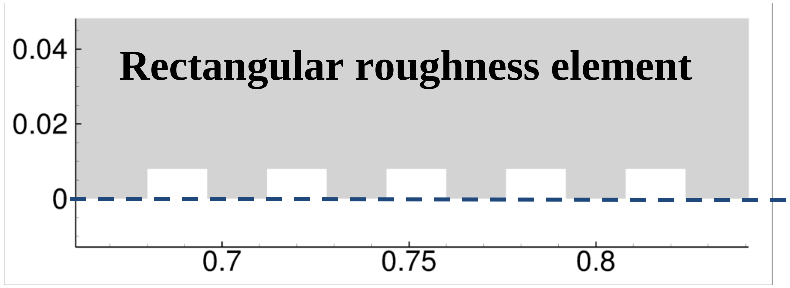
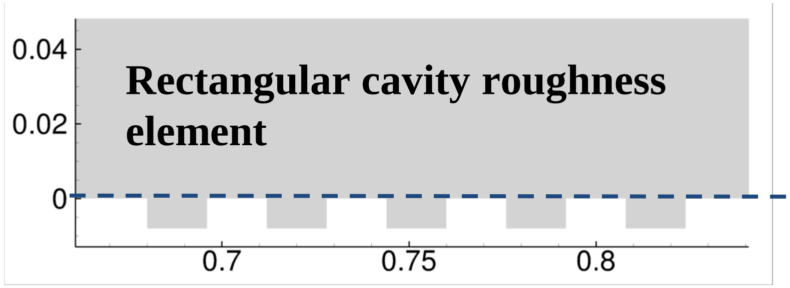
Wall Roughness Set-Up





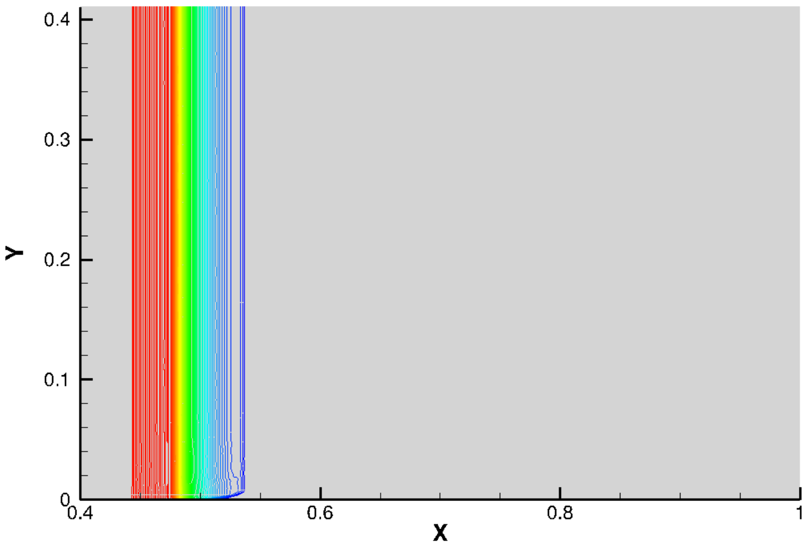


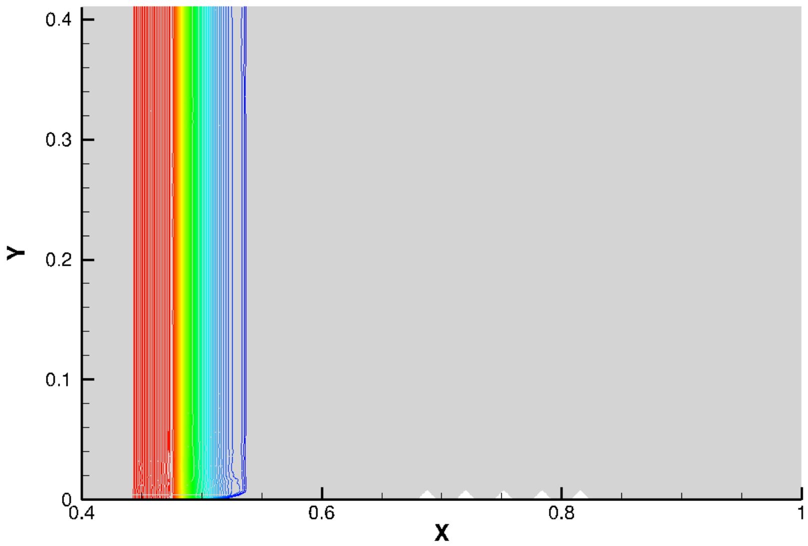
H/D	Triangular Element	Rectangular Element	Rectangular Cavity Element
1/32	Tri_H1	-	-
1/16	Tri_H2	-	-
1/8	Tri_H3	-	-
1/4	Tri_H4	Rec_H4	Rec_Cav_H4
1/2	-	-	Rec_Cav_H5

Cases summary

For all cases, the Reynolds number is $Re = 1{,}000$

SBLI over Triangular Surface Elements



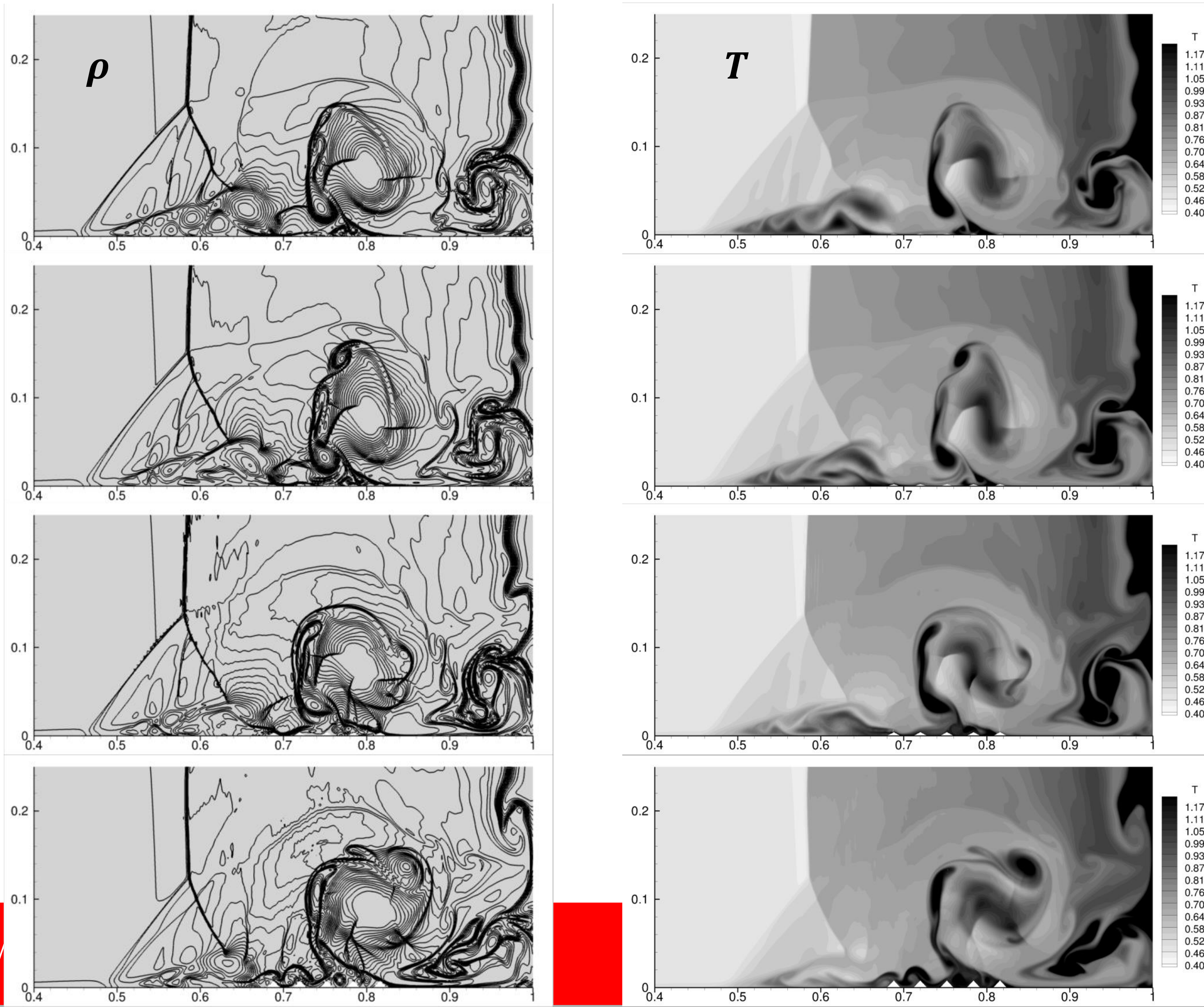


Tri_H1

Tri_H4

Density fields

Flow Fields Comparison of Different *H/D*



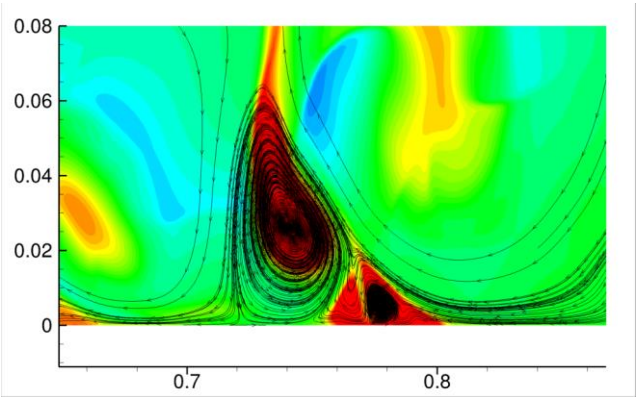
$$\frac{H}{D} = \frac{1}{32}$$

$$\frac{H}{D} = \frac{1}{16}$$

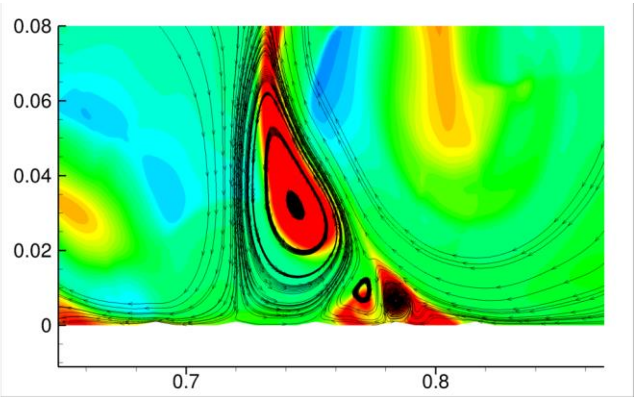
$$\frac{H}{D} = \frac{1}{8}$$

$$\frac{H}{D} = \frac{1}{4}$$

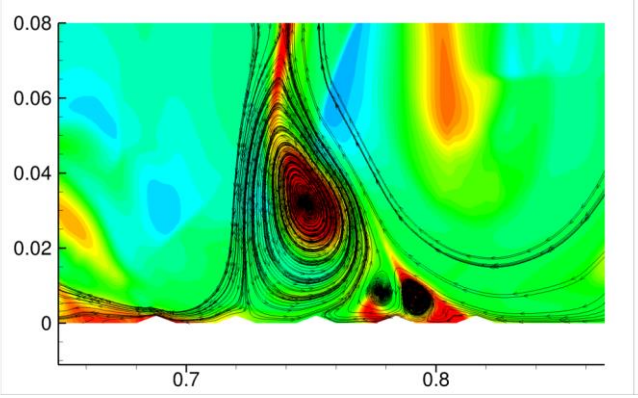
Streamlines & Temperature near Roughness



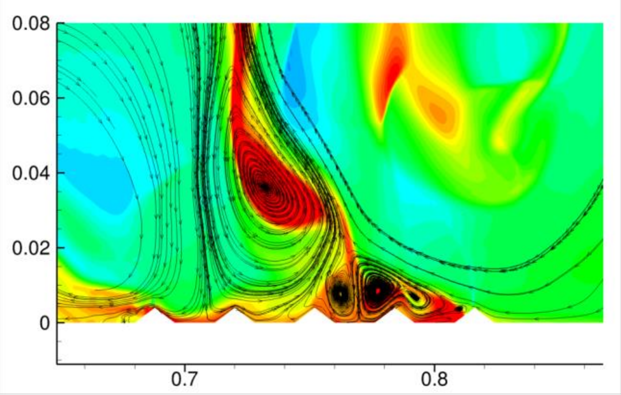
Smooth



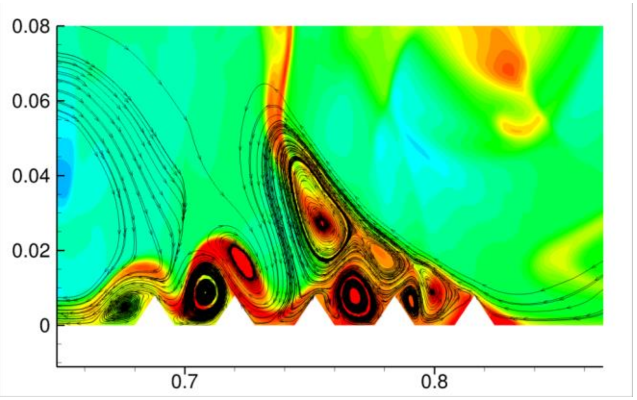
$H/D = 1/32$



$H/D = 1/16$

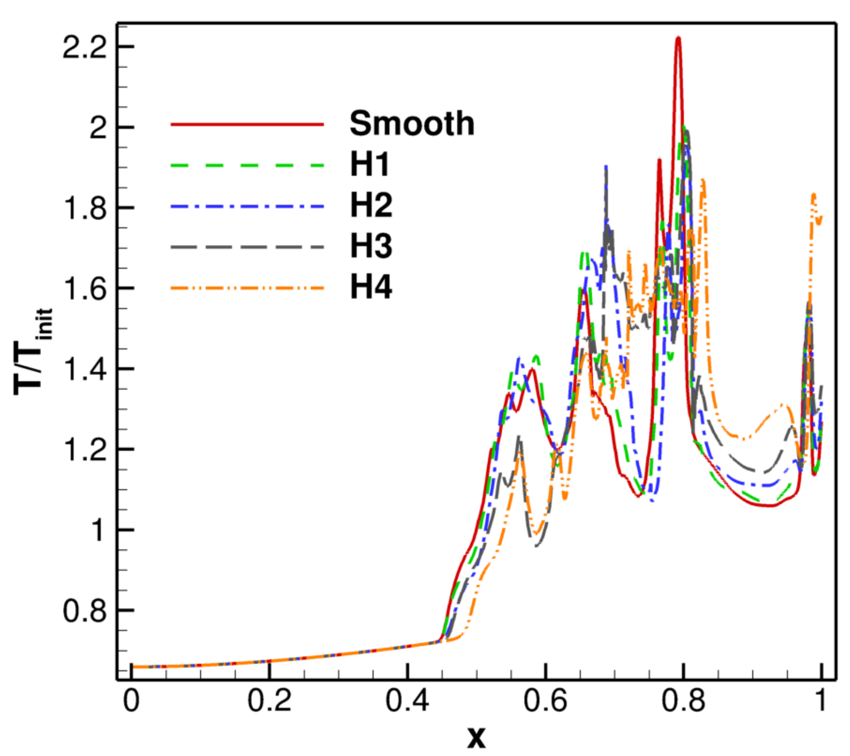


$H/D = 1/8$

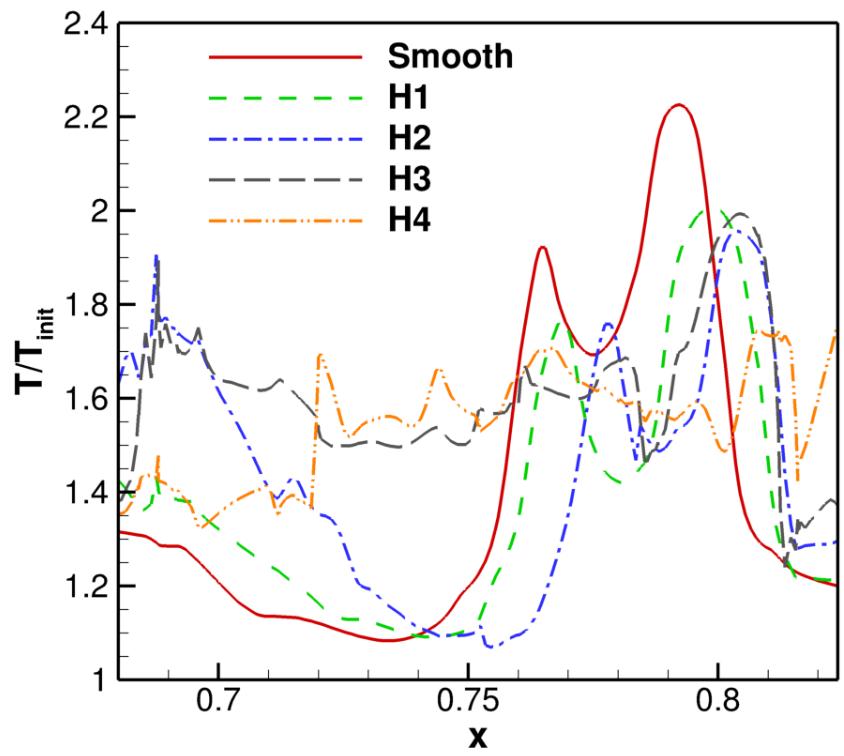


$H/D = 1/4$

Wall Temperature Comparison of Different *H/D*

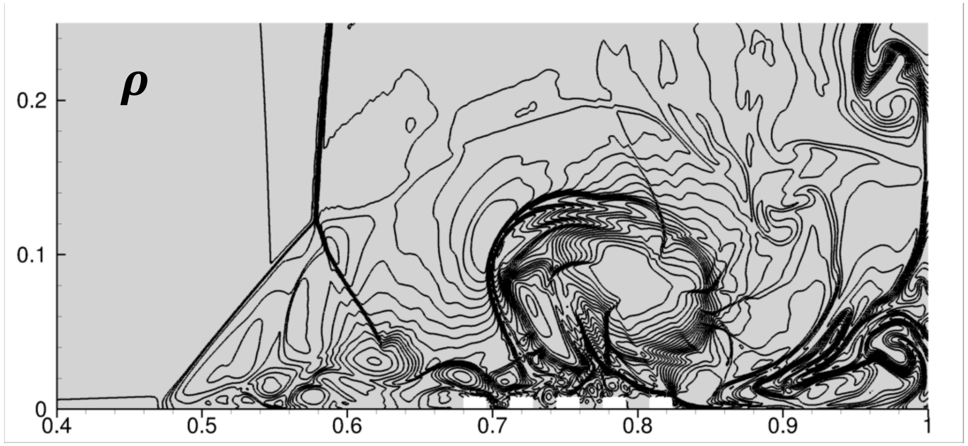


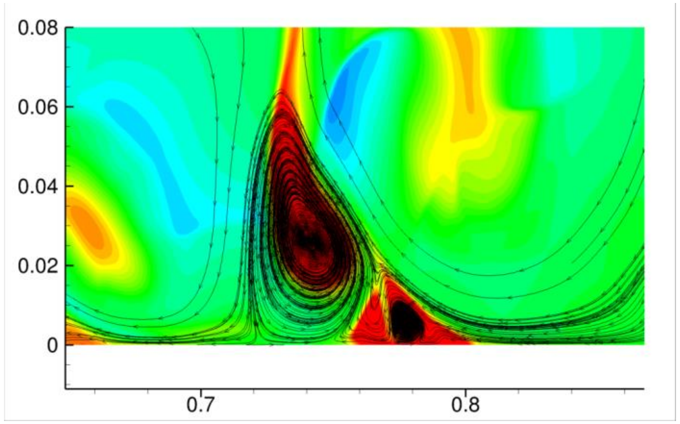
Overview of wall temperature



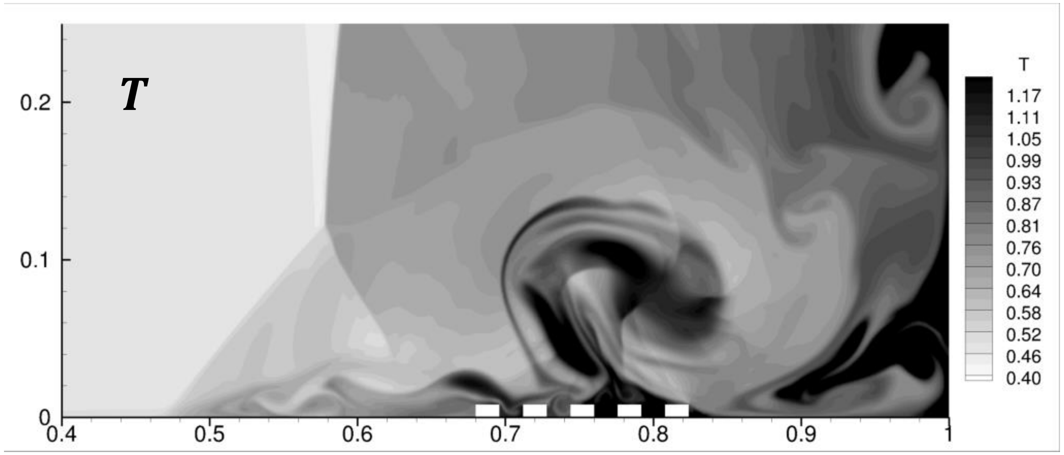
Close-up view of wall temperature

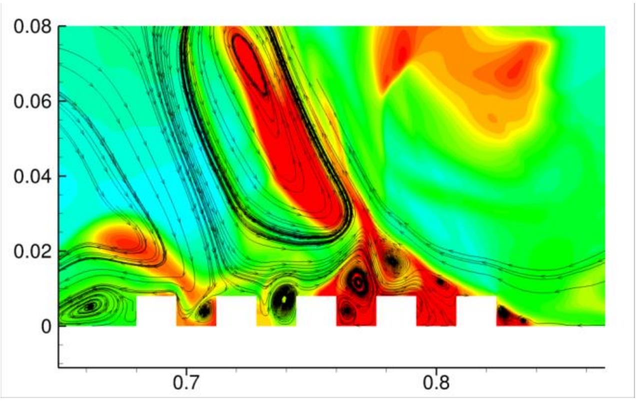
Flow Fields over Rectangular Roughness





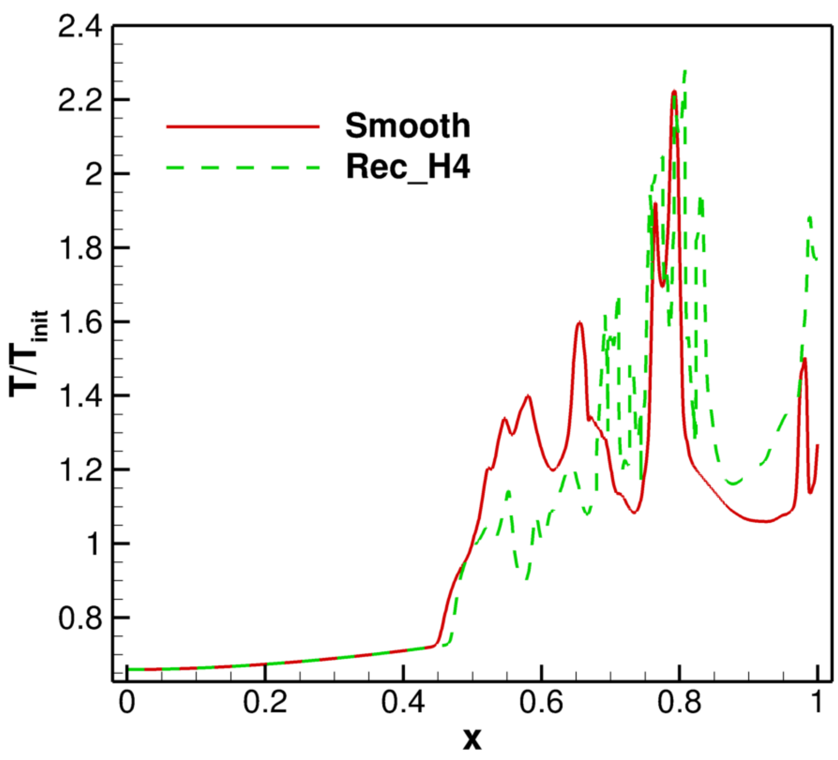
Smooth



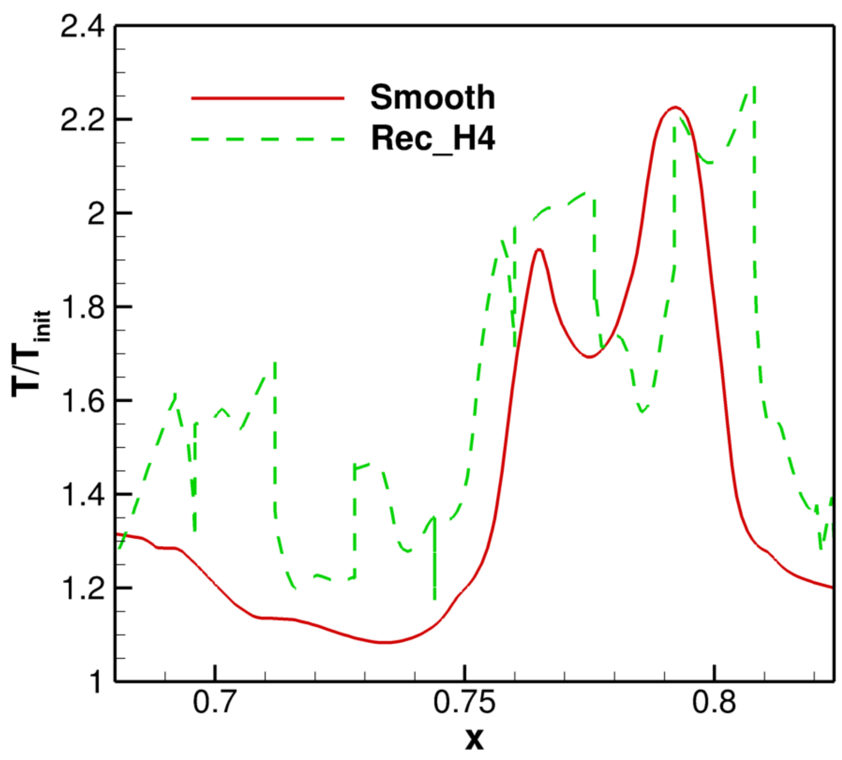


$H/D = 1/4$

Wall Temperature Comparison of Different *H/D*

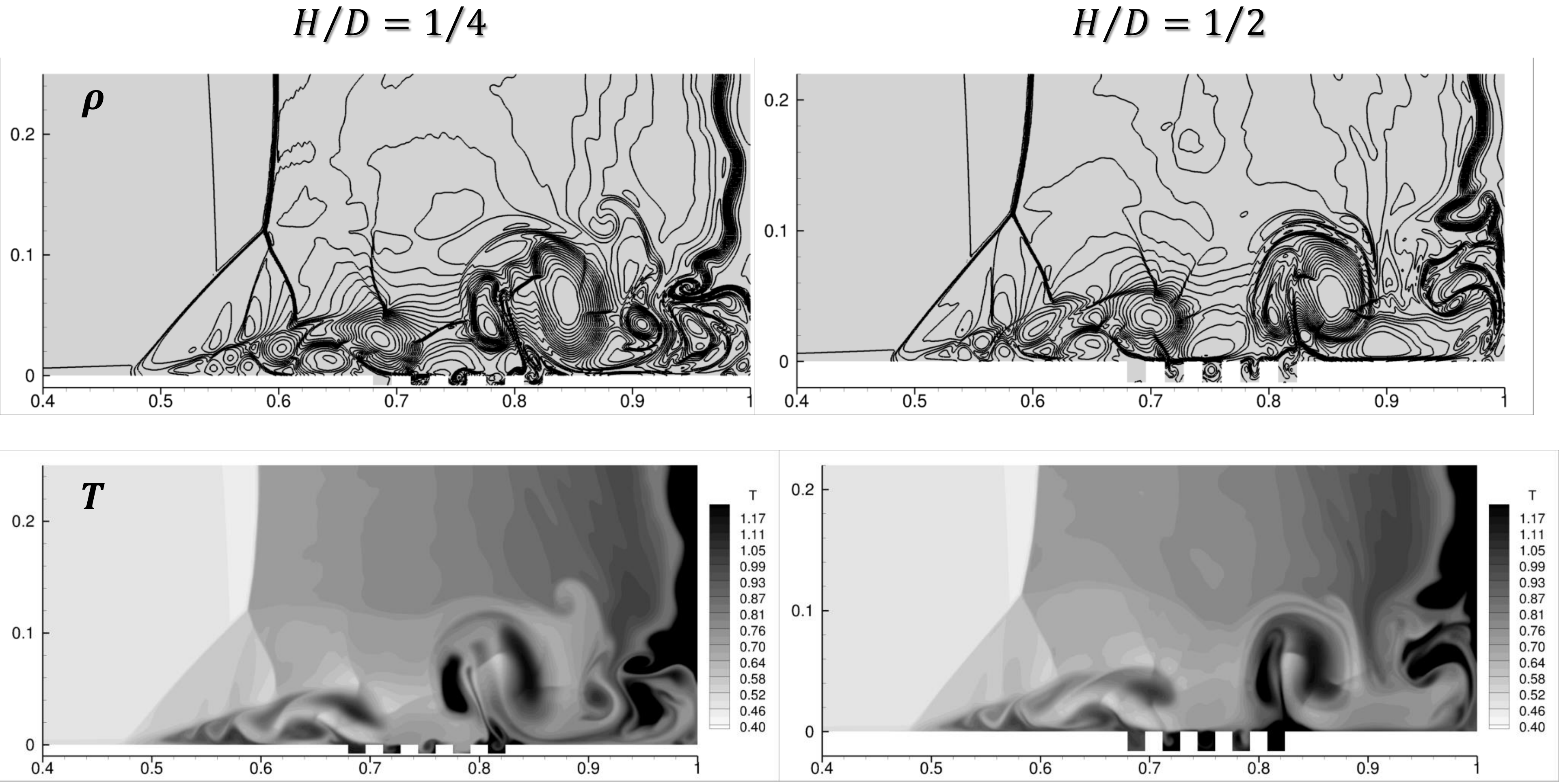


Overview of wall temperature

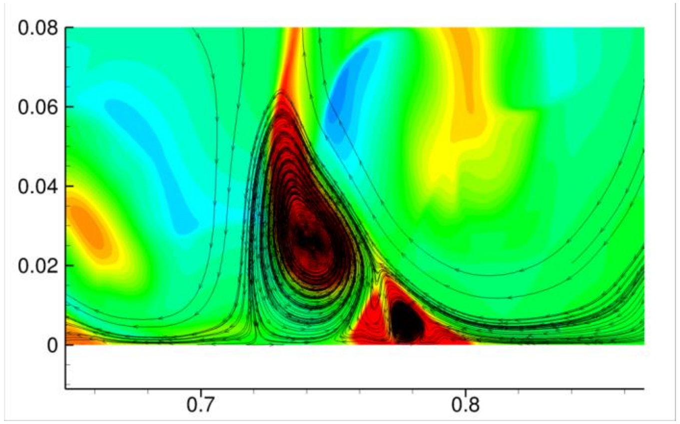


Close-up view of wall temperature

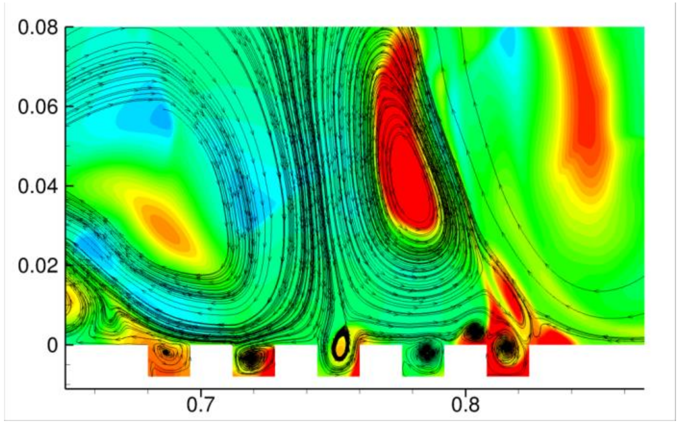
Flow Fields over Rectangular Cavity Roughness



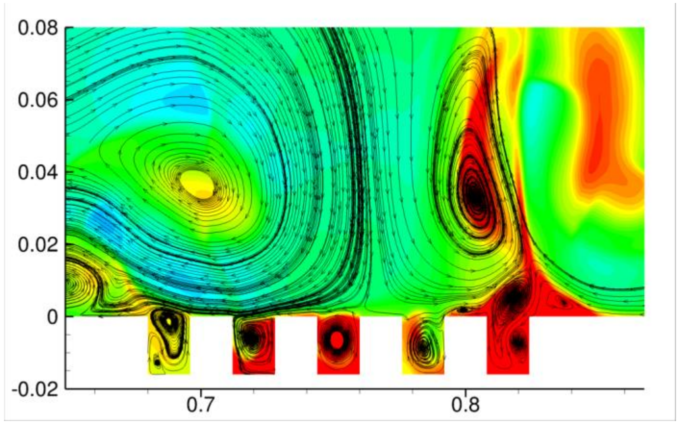
Streamlines & Temperature near Roughness



Smooth

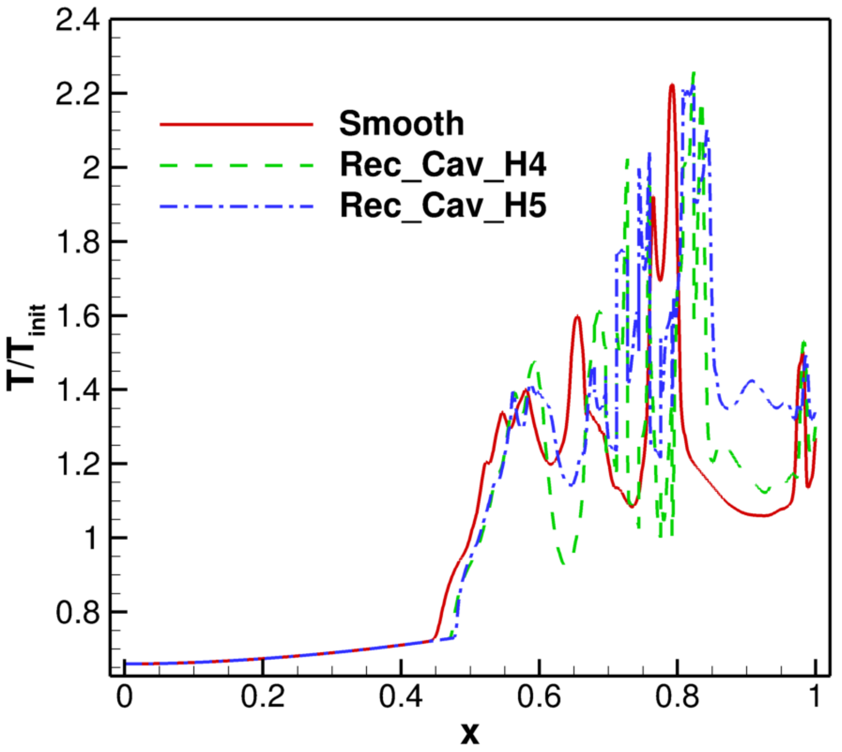


$H/D = 1/4$

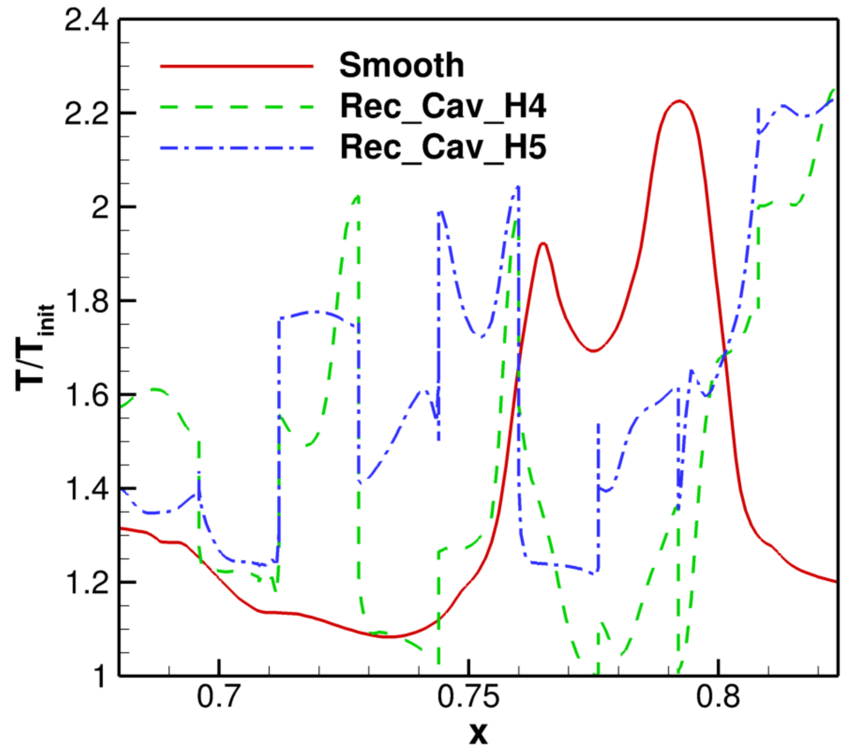


$H/D = 1/2$

Wall Temperature Comparison of Different *H/D*

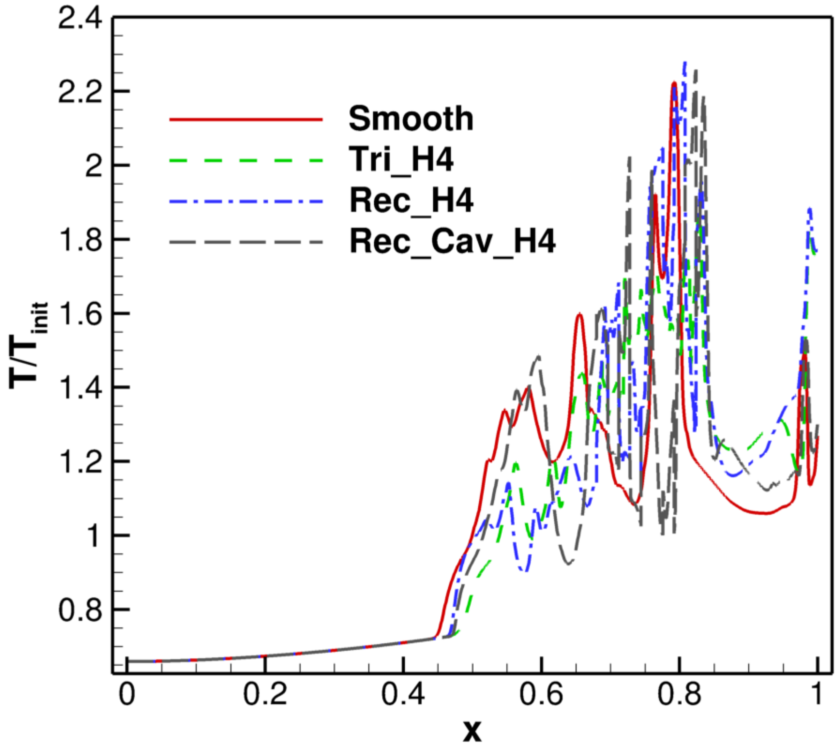


Overview of wall temperature

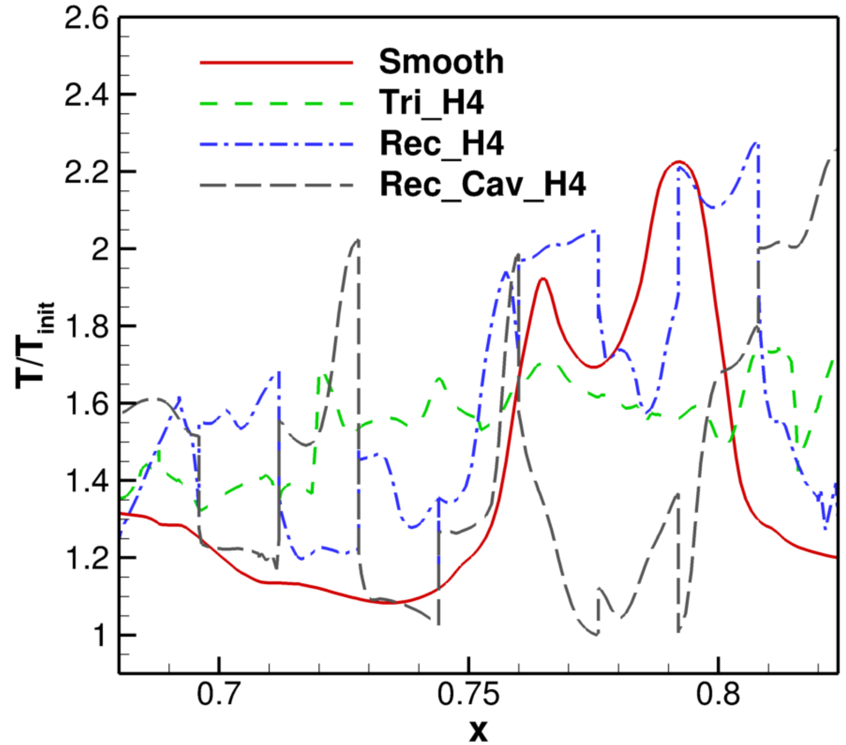


Close-up view of wall temperature

Wall Temperature Comparison of Different Types of Roughness Elements



Overview of wall temperature



Close-up view of wall temperature

Contents

- Background
- Numerical Methods & Verification
- Shock/Boundary Layer Interaction (SBLI)
  - SBLI over smooth walls
  - SBLI over surface roughness
- **Conclusions & Future Work**

Conclusions & Future Work

- A localized Laplacian artificial viscosity (LLAV) stabilization procedure is developed for the high-order unstructured-grid-based flux reconstruction/correction procedure via reconstruction (FR/CPR) method.

- The FR/CPR-LLAV method is used to simulate shock-boundary layer interaction (SBLI) with and without wall roughness.
  - The FR/CPR-LLAV method can sharply capture shock structures and efficiently resolve boundary-layer separation features
  - The FR/CPR-LLAV method is capable of flow simulation over complex geometry
  - The FR/CPR-LLAV method is compact, and therefore, is suitable for parallel computing

Conclusions & Future Work (Cont.)

- Effects of surface roughness on SBLI are numerically investigated.
  - Surface roughness can substantially modify the interaction between the shock and the boundary layer, thus affecting surface heat transfer processes
  - In the current 2D study, the triangular roughness elements with relatively large height-width ratio can enhance the mixing near the wall
  - The rectangular (cavity) roughness elements with relatively large height-width ratio can trap the evolving separation vortices, resulting in redistribution of surface temperature

- More studies on 2D roughness elements of different types
- Extension to 3D shock-turbulence interaction

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Thank you!